Module - 5

Digital Representation of Analog Signals

Why Digitize Analog Source?

* Digital systems are less sensitive to noise than analog
* With digital systems, it is easier to integrate different services, for example, video and the accompanying soundtrack, into the same transmission scheme.
* Various media sharing strategies, known as multiplexing techniques, are more easily implemented with digital transmission strategies.
* Analog data cannot be analyzed on Digital computer.
* Analog data cannot be compressed efficiently.
* Analog data transmission do not have efficient error detection and correction techniques.
* Circuitry for handling digital signals is easier to repeat and digital circuits are less sensitive to physical effects such as vibration and temperature.

Sampling:

- To convert an analog signal into digital data. Sampling, quantization and encoding has to be performed.
- An analog signal is first sampled, which can be defined as "the process of converting an analog signal into discreet time signal by measuring the signals at periodic instants of time".

Sampling theorem:

Statement:

"If a finite energy signal $g(t)$ contains no frequencies higher than 'W' hertz, it can be completely recovered from its ordinates of a sequence points spaced '$1/2W$' seconds apart".

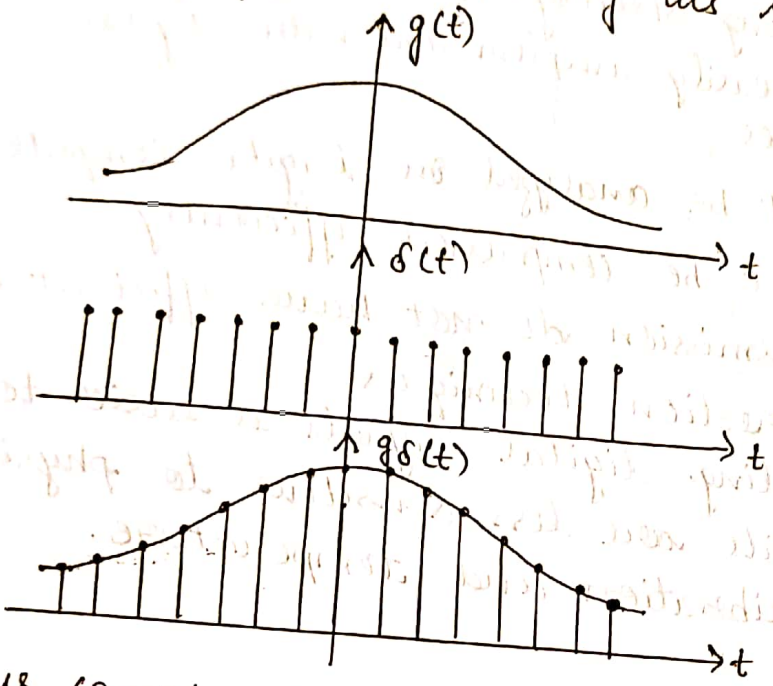
Proof:

The sampling theorem for band limited signal can be proved in two parts.

1) Representing $g(t)$ in terms of its samples
2) Reconstructing it from its samples.

To prove this, let us consider an analog signal $g(t)$ of finite energy and infinite duration as shown in figure.

Representing $g(t)$ in terms of its samples $g(nT_s)$



Let us consider the sample values of $g(t)$ at $t = 0, \pm T_s, \pm 2T_s, \pm 3T_s$ .... Therefore the entire series can be denoted as $g(nT_s)$, where $n = 0, \pm 1, \pm 2, \pm 3$ ....

where T_s = sampling period, $f_s = \frac{1}{T_s}$ = sampling rate

To obtain $g_\delta(t)$ from $g(t)$, we need to multiply $g(t)$ with δ function.

$$\left\langle g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t - nT_s) \right\rangle \longrightarrow (1)$$

where, $g(nT_s)$ is samples of $g(t)$

$\delta(t - nT_s)$ indicates the samples placed at $\pm T_s$, $\pm 2T_s$, $\pm 3T_s$ .... & so on.

(1) is graphically represented in above figure

Since $g(nT_s)$ are samples of $g(t)$

$$g(t)\,\delta(t-nT_s) = g(nT_s)\,\delta(t-nT_s)$$

Eqn (1) can be written as,

$$g_\delta(t) = g(t)\sum_{n=-\infty}^{\infty}\delta(t-nT_s) \longrightarrow (2)$$

W.K.T multiplication of 2 time function is equivalent to the convolution of their respective fourier transforms

$$g(t) \rightarrow G(f)$$

$$g_\delta(t) \rightarrow G_\delta(f)$$

$$\delta(t-nT_s) \rightarrow f_s\sum_{n=-\infty}^{\infty}\delta(f-nf_s)$$

$$\therefore\ G_\delta(f) = FT\{g(t) * FT\,\delta(t-nT_s)\}$$

$$G_\delta(f) = G(f) * f_s\sum_{n=-\infty}^{\infty}\delta(f-nf_s)$$

$$\left\langle G_\delta(f) = f_s\sum_{n=-\infty}^{\infty} G(f) * \delta(f-nf_s)\right\rangle \longrightarrow (3)$$

From the properties of delta function, Eqn (3) becomes

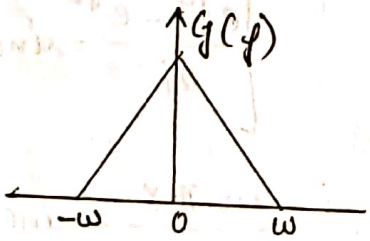
$$\left\langle G_\delta(f) = f_s\underbrace{\sum_{n=-\infty}^{\infty} G(f-nf_s)}_{g(t)}\right\rangle \longrightarrow (4)$$

The spectrum of the above expression assuming $f_s = 2w$ and we know that $\delta(t-nT_s) \rightarrow e^{-j2\pi nfT_s}$

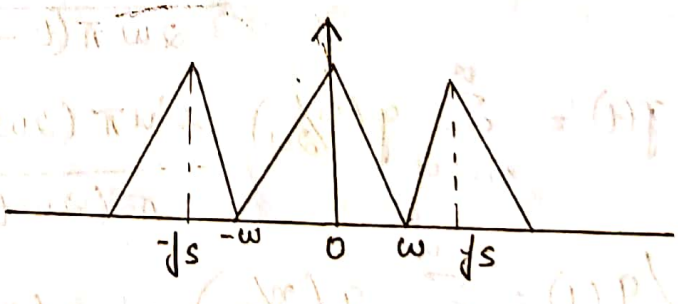
$$= e^{-j\omega nT_s}$$

Taking FT of Eqn (1), becomes

$$\left\langle G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s)\,e^{-j2\pi nfT_s}\right\rangle \longrightarrow (5)$$



Spectrum of $g(t)$



Spectrum of $g_\delta(t)$ for $f_s = 2w$

Reconstruction of signal from it's samples

Let us consider $f_s = 2w$, $T_s = 1/2w$ $\quad [T_s = 1/f_s = 1/2w]$

$\therefore$ Eqn (5) implies

$$\left\langle G_\delta(f) = \sum_{n=-\infty}^{\infty} g(n/2w)\,e^{(-j\pi nf/w)}\right\rangle \longrightarrow (6)$$

substituting $f_s = 2w$ in Eq^n (4)

$$G_\delta(f) = f_s\, G(f)$$

$$G_\delta(f) = 2w\, G(f)$$

$$\therefore \left\langle G(f) = \tfrac{1}{2w}\, G_\delta(f) \right\rangle \rightarrow (7)$$

substituting Eq^n (6) in (7)

$$\left\langle G(f) = \frac{1}{2w} \sum_{n=-\infty}^{\infty} g(n/2w)\, e^{-j\pi n f/w} \right\rangle \rightarrow (8)$$

Taking inverse FT of $G(f)$

$$\left\langle g(t) = \int_{-\infty}^{\infty} G(f)\, e^{j2\pi f t}\, df \right\rangle \rightarrow (9)$$

$$g(t) = \int_{-\infty}^{\infty} \frac{1}{2w} \sum_{n=-\infty}^{\infty} g(n/2w)\, e^{-j\pi n f/w}\, e^{j2\pi f t}\, df$$

Interchange the order of summation and Integration

$$g(t) = \sum_{n=-\infty}^{\infty} g(n/2w)\, \frac{1}{2w} \int_{-w}^{w} e^{j2\pi f(t - n/2w)}$$

$$g(t) = \sum_{n=-\infty}^{\infty} g(n/2w)\, 1/2w \left[\frac{e^{j2\pi f(t-n/2w)}}{j2\pi(t-n/2w)} \right]_{-w}^{w} \qquad \int e^{af} = \frac{e^{af}}{a}$$

$$g(t) = \sum_{n=-\infty}^{\infty} g(n/2w)\, 1/2w \left[\frac{e^{j2\pi w(t-n/2w)} - e^{-j2\pi w(t-n/2w)}}{j2\pi(t-n/2w)} \right]$$

$$g(t) = \sum_{n=-\infty}^{\infty} g(n/2w)\, \frac{\sin 2\pi w(t-n/2w)}{2w\pi(t-n/2w)} \qquad \left[\frac{e^{j\theta} - e^{-j\theta}}{2j} = \sin\theta \right]$$

$$g(t) = \sum_{n=-\infty}^{\infty} g(n/2w)\, \frac{\sin\pi(2wt-n)}{\pi(2wt-n)} \qquad \frac{\sin\pi x}{\pi x} = \text{sinc}$$

$$\left\langle g(t) = \sum_{n=-\infty}^{\infty} g(n/2w)\, \text{sinc}(2wt-n) \right\rangle \rightarrow (10)$$

Eq^n (10) is called an interpolation formula for reconstruction of the original signal $g(t)$.

A continuous time signal can be completely represented in its samples and recovered back if the sampling frequency is twice of the highest frequency content of the signal i.e

$$f_s \geq 2W$$

where, f_s = sampling frequency
W = Higher frequency content of message signal.

ii) <u>Nyquist rate</u>:

When the sampling rate becomes exactly equal to $2W$ samples/sec for a given Bandwidth of W Hertz then it is called Nyquist rate.

$$\text{Nyquist rate} = 2W \text{ Hz}.$$

<u>Nyquist Interval</u>:

It is the time interval between any two adjacent samples when sampling rate is Nyquist rate.

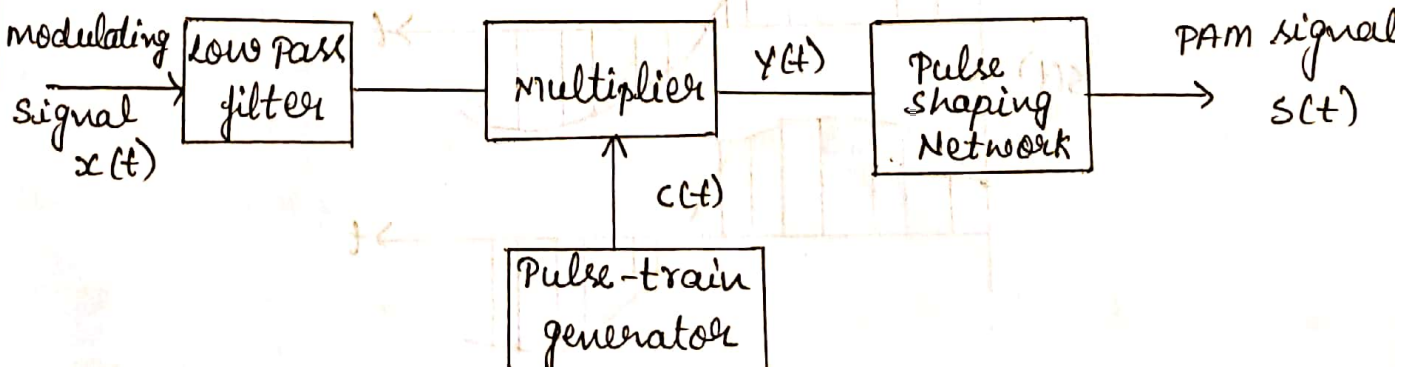
$$\text{Nyquist interval} = \frac{1}{2W} \text{ seconds}.$$

Pulse Amplitude Modulation:

The process in which the amplitude of the pulse is varied with respect to amplitude of the modulating signal at the sampling instant keeping width and position of pulse constant is known as pulse amplitude Modulation (PAM).

<u>Generation of PAM</u>:

Block diagram for PAM generation is shown in figure



- The modulating signal $x(t)$ is bandlimited to freq W Hz by passing through LPF of frequency W Hz. Low pass filtering also avoids aliasing.
- The bandlimited signal is then sampled at the multiplier. Multiplier samples $x(t)$ with the help of pulse-train generator signal $c(t)$ as shown in figure. Multiplication of $x(t)$ and $c(t)$ produces PAM signal $Y(t)$.
- The pulse shaping Network produces the flat top pulses as shown in fig. This is the required PAM signal.

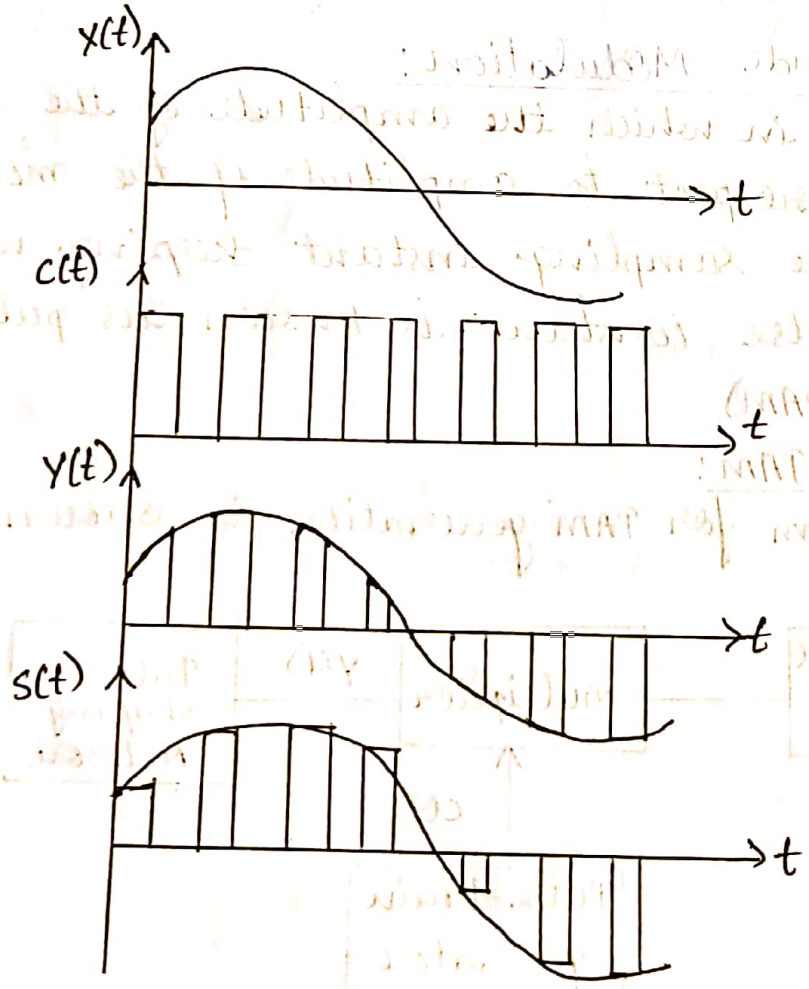
Note: <u>Flat top sampling:</u>

The top of the samples remain constant and it is equal to instantaneous value of baseband signal $x(t)$ at the start of sampling.

By using flat-top samples to generate a PAM signal, amplitude distortion occurs.

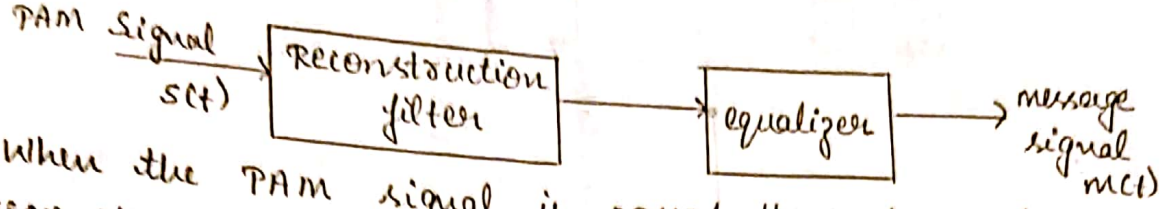
The distortion caused by the use of PAM to transmit an analog information bearing signal is referred to as the aperture effect.

<u>Waveforms:</u>



...ction of PAM:

Recovering of m(t) from PAM signal s(t) is known as detection.



- When the PAM signal is passed through a lowpass reconstruction filter, the filter reconstructs the analog signal from PAM pulses.
- Equalizer in cascade with lowpass reconstruction filter compensates the aperature effect and produces the message signal m(t).

Advantages:

- PAM can be easily generated and detected.

Disadvantages:

- Bandwidth required for PAM transmission is larger than the maximum frequency of message signal [i.e Whz]
- Interface of noise is maximum, because the amplitud of PAM pulses varies according to message signal.

Applications:

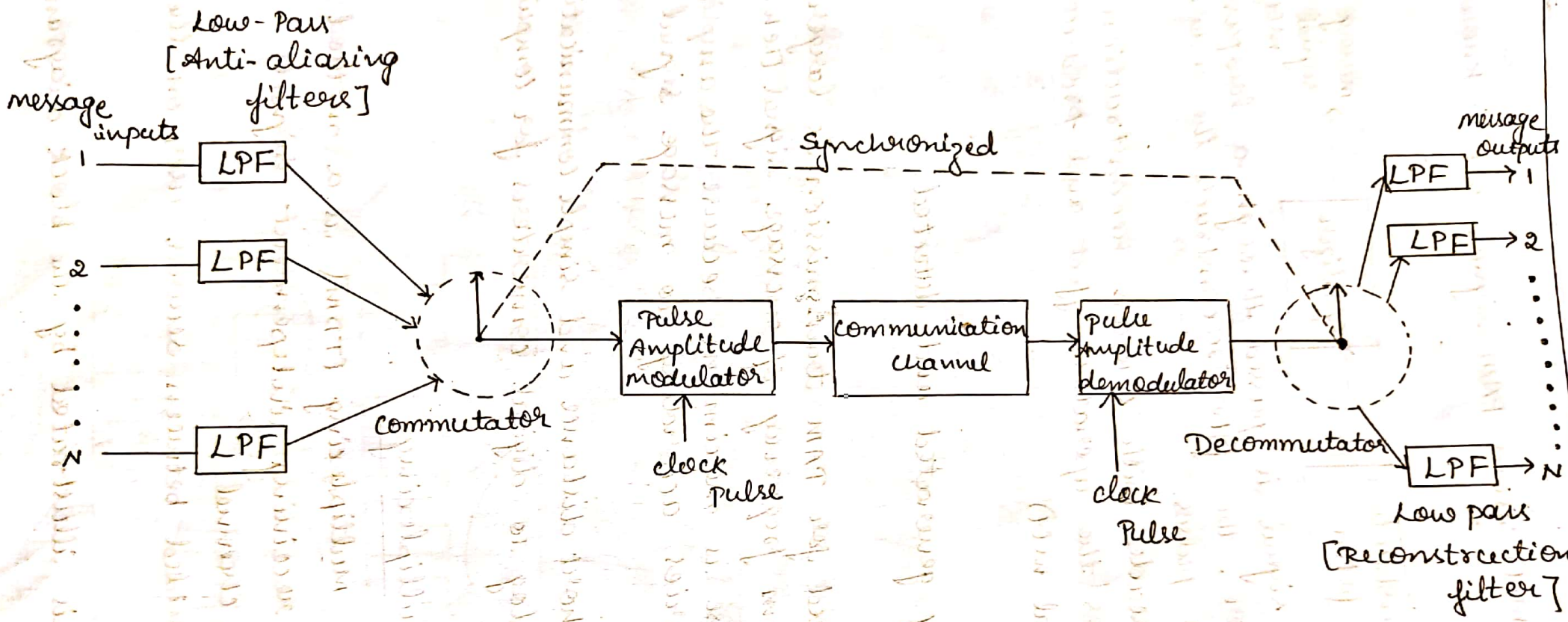
- PAM is used for short distance and simple communication
- It is used as Analog to digital converters for compute interfacing.

Time Division Multiplexing:

Time Division Multiplexing [TDM] is a method transmitting and receiving independent signals over a common channel.

channel is divided between several users on the basis of time.

The concept of TDM is illustrated by the block diagra shown in figure.



Block Diagram of TDM System

Each input signal is first restricted in Bandwidth by a low-pass filter to remove the frequencies that are non essential.

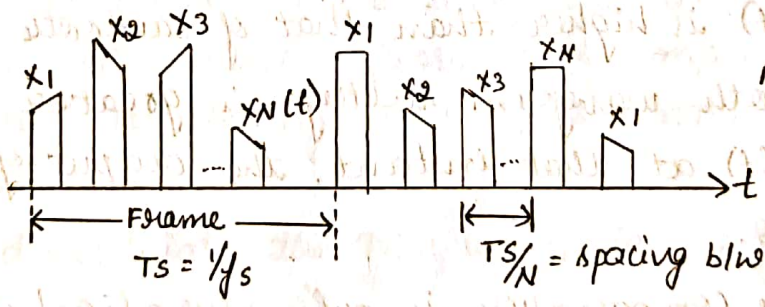
- The pre-aliasing filter output are then applied to a commutator which consists of electronic switch.

The functions of the commutator is

1) To take a narrow samples of each of the N input messages at a rate f_s that is slightly higher than 2W.
2) To sequentially interleave these N samples inside the sampling interval $T_s = 1/f_s$.

- After the commutation process, the multiplexed signal is applied to a pulse modulator, the purpose of which is to transform the multiplexed signal into a form suitable for transmission over the common channel.
- At the receiving end, pulse amplitude demodulator is used to perform the reverse operation of PAM.
- Decommutator picks the samples of incoming signal and distributes to appropriate low-pass reconstruction filter. Decommutator operates in synchronism with the commutator in the transmitter.

Note:



multiplexed PAM wave

- Spacing between two pulses $= T_s/N$
- Signalling rate / transmission rate / bit rate $r = \dfrac{1}{\text{spacing b/w two pulses}}$

$$r = 1/(T_s/N)$$

$$\boxed{r = N f_s}$$

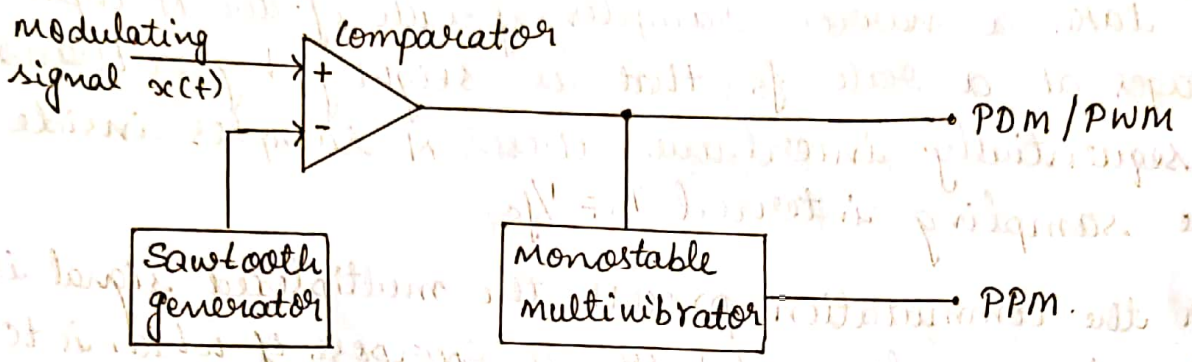
- Minimum transmission Bandwidth of TDM channel is

$$(BW)_T = NW$$

Pulse Position Modulation:

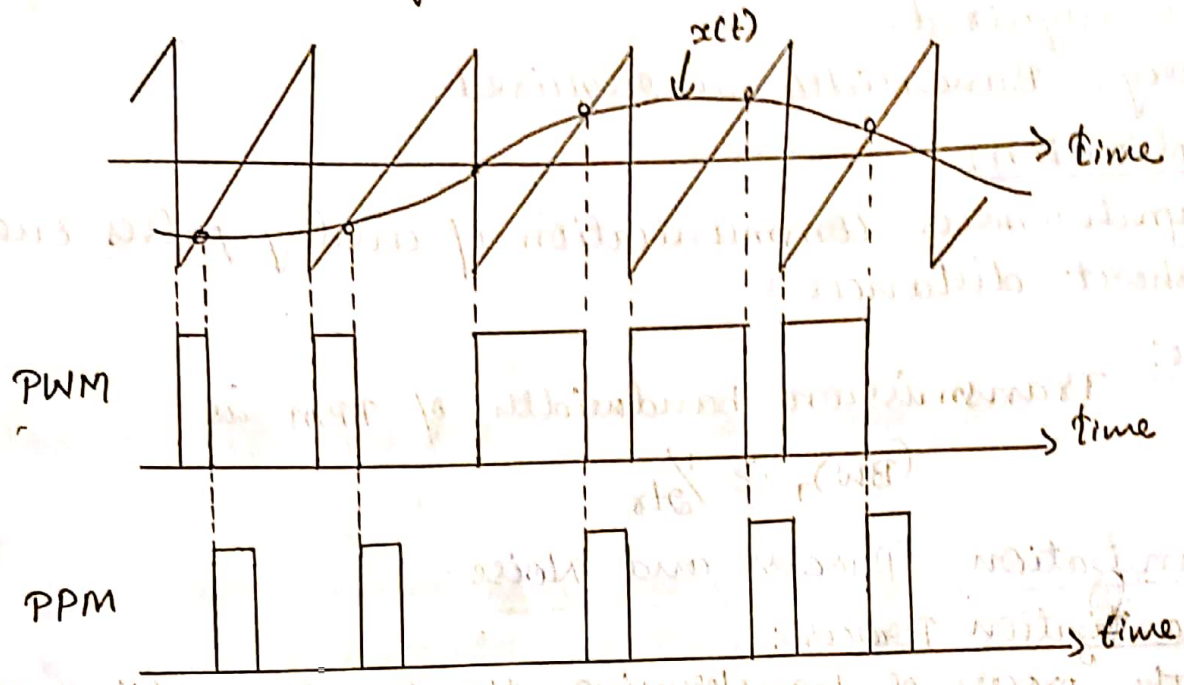
It is a type of pulse modulation, in which the position of each pulse is varied with respect to the amplitude of modulating signal keeping amplitude and width of the pulse constant.

Generation of PPM:



- The block diagram to generate PPM is shown in above figure. The scheme combines both sampling and modulation operation.
- Sawtooth generator generates sawtooth signal of frequency fs and it is applied to the inverting input of comparator.
- Modulating signal x(t) is applied to the non-inverting input of the comparator as shown in figure.
- Output of the comparator is high when instantaneous value of x(t) is higher than that of sawtooth waveform.
- When sawtooth waveform voltage is greater than voltage of x(t) at that instant, the output of comparator remains zero.
- Output of the comparator is pulse duration/ width modulation as shown in waveform.
- To generate PPM, PDM / PWM signal is used as the trigger input to one monostable multivibrator.
- The monostable output remains zero until it is triggered the monostable is triggered on the falling edge of PDM. the output of monostable then switches to positive

saturation level. This voltage remains high for the fixed period then goes low.



Detection of PPM waves:

To get back original signal m(t) from s(t) PPM receiver may proceed as follows:

- Convert the received PPM wave into PWM wave with the same modulation.
- Integrate this PWM wave using a device with a finite integration time.
- Sample the output of the integrator at a uniform rate to produce a PAM wave, whose pulse amplitudes are proportional to the signal samples $x(nT_s)$ of the original PPM wave s(t).
- Finally, demodulate the PAM wave to recover the message signal m(t).

Advantages:

- In PWM amplitude is held constant thus less noise interference.
- Signal and noise seperation is very easy.
- Due to constant pulse widths and amplitudes, transmission power for each pulse is same.

Disadvantages:

- Synchronization between transmitter and receiver is required.
- Large Bandwidth is required.

Application:

- Synchronous communication of analog pulses over short distances.

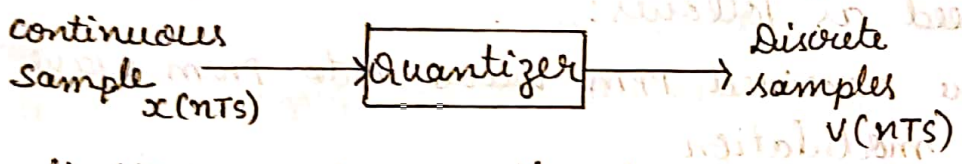
Note: Transmission bandwidth of PPM is

$$(BW)_T \geq 1/2t_r$$

Quantization Process and Noise:

Quantization Process:

The process of transforming the sample amplitude $x(nT_s)$ of a message signal $x(t)$ into a discrete amplitude $V(nT_s)$ value is referred as quantizing process.



Quantization can be broadly classified into uniform quantization and non uniform quantization.

In uniform quantization the difference between two quantization levels (step size) remains constant over the complete amplitude range otherwise it is non-uniform quantization.

Types of uniform Quantizers:

1) Mid-Rise type Quantizer
2) Mid-Tread type Quantizer.

In the stair case like graph, the origin lies the middle of the tread portion in mid-Tread type where as the origin lies in the middle of the rise portion in the mid-Rise type.

An anti-aliasing filter is basically a filter used to ensure that the input signal to sampler is free from the unwanted frequency components. LPF remove the frequencies greater than 'w' before sampling.

Sampling:

The incoming message signal is sampled by passing through the sampler. In order to ensure perfect reconstruction of the message signal at the receiver, the sampling rate must be greater than twice the highest frequency components 'w' of the message signal.

Quantization:

The process of transforming sampled amplitude values of a message signal into a discrete amplitude value is referred to as Quantization.

Encoding:

Even after sampling and quantizing, discrete set of values are not in the form best suited to transmission over a line or radio path. Thus to make the signal more robust to noise, interference and other channel degradations, Encoding has to be carried out.

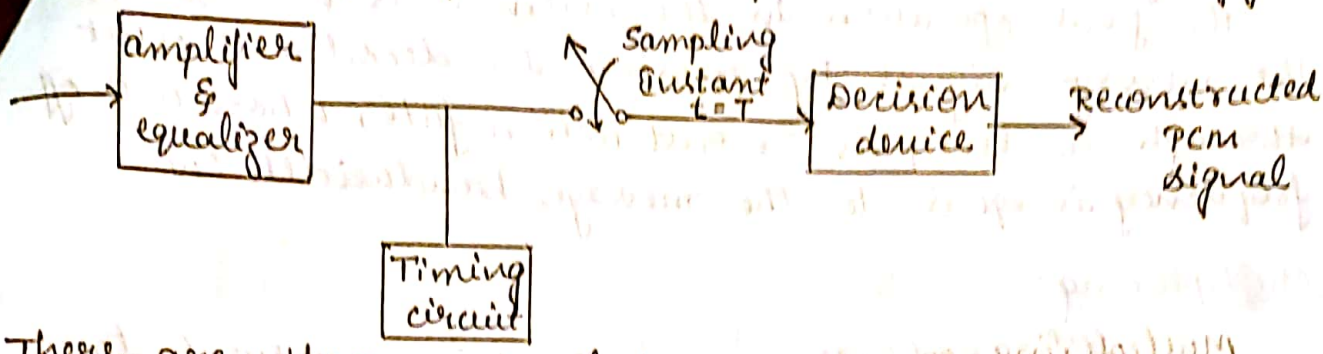
Encoding process translates the discrete set of sample value to a more appropriate form of binary signal.

Regenerative Repeater:

When the PCM signal is transmitted over the channel, it get distorted due to channel noise. Hence regenerative repeater are used at regular spacings to reconstruct the PCM signal back.

Regenerative repeaters receives the noisy PCM signal, performs amplification and equalization on it and constructs new PCM signal.

ock diagram of regenerative repeater is shown in figure



There are three main blocks
1) Equalizer and amplifier
2) Timing circuit
3) Decision Device

The distorted PCM signal is amplified by amplifier. The equalizer shapes the received pulses so as to compensate for the effects of amplitude and phase distortion produced by the transmission characteristics of the channel.
The timing circuit derives the sampling instant t=T, where the signal to noise ratio is maximum.
The decision device compares the sampled PCM signal with threshold. If the threshold is exceeded, the new binary '1' is generated, otherwise binary '0' is sent. Thus the new PCM signal is generated, which is totally free of noise.

Problems associated with regenerative repeater.

1) If the channel noise is too high, then binary '1' is treated as binary '0' and vice-versa.
2) The spacing between the pulses is not exactly same as that was transmitted. This is called timing jitter. It creates distortion in the regenerated PCM signal.

Decoding:

The first operation in the receiver is to regenerate the received pulses one last time. These pulses are then regrouped into codewords and decoded into a quantized PAM signal.

Filtering:

The final operation in the receiver is to recover the message signal by passing the decoder output through a low pass reconstruction filter whose cut-off frequency is equal to the message bandwidth 'W'.

Multiplexing:

The applications using PCM, it is natural to multiplex different message sources by time division.

As the number of independent message sources is increased, the time interval that may be alloted to each source has to be reduced, since all of them must be accomodated into a time interval equal to the reciprocal of the sampling rate.

Quantization Noise and signal to Noise ratio (SNR) in PCM:

Error introduced because of quantization is called quantization error.

$$\langle \varepsilon = x(nT_s) - v(nT_s) \rangle$$

The signal to quantization noise ratio is defined as

$$\frac{S}{N} = \frac{\text{average power of the signal}}{\text{average power of the quantization noise}}$$

average power of quantization Noise:

Since quantization noise 'ε' is random variable, average power of quantization noise is $E[\varepsilon^2]$ mean square value [If $R=1$].

W.K.T The mean square value of a random variable 'x' is

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f_x(x)\,dx$$

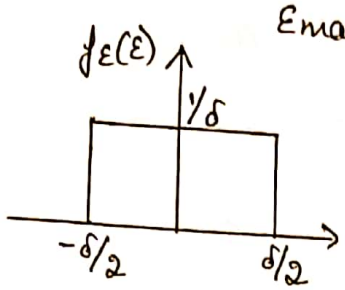
III^{ly}

$$E[\varepsilon^2] = \int_{-\infty}^{\infty} \varepsilon^2 f_\varepsilon(\varepsilon)\,d\varepsilon$$

Assume that the quantization noise 'ε' will be uniformly distributed random variable over the interval $[-\delta/2, \delta/2]$

The maximum quantization noise is

$$\varepsilon_{max} = |\delta/2| = \pm \delta/2$$



$$f_\varepsilon(\varepsilon) = \begin{cases} 1/\delta & -\delta/2 \le \varepsilon \le \delta/2 \\ 0 & \varepsilon > \delta/2 \end{cases}$$

$$E[\varepsilon^2] = \int_{-\infty}^{\infty} \varepsilon^2 f_\varepsilon(\varepsilon)\, d\varepsilon$$

$$= \int_{-\delta/2}^{\delta/2} \varepsilon^2 \, 1/\delta \, d\varepsilon = 1/\delta \int_{-\delta/2}^{\delta/2} \varepsilon^2 \, d\varepsilon$$

$$= \frac{1}{\delta} \left. \frac{\varepsilon^3}{3} \right]_{-\delta/2}^{\delta/2} = \frac{1}{3\delta}\left[\frac{\delta^3}{8} + \frac{\delta^3}{8}\right]$$

$$E[\varepsilon^2] = \frac{1}{3\delta}\left[\frac{\delta^3}{4}\right] = \frac{\delta^2}{12}$$

$$\boxed{E[\varepsilon^2] = \frac{\delta^2}{12}}$$

Thus $S/N = \dfrac{P}{\delta^2/12}$

$$\left(\frac{S}{N}\right) = \frac{12P}{\delta^2}$$ where, δ = step size

Let $x(nT_s)$ be of continous sample amplitude ranges from $-x_{max}$ to $+x_{max}$.

$\therefore$ Total amplitude range $= x_{max} - (-x_{max}) = 2x_{max}$

If this amplitude range is divided into 'q' levels of quantizer, then the step size 'δ' is given as

$$\delta = \frac{2x_{max}}{q}$$

For Normalised signal

$$\delta = 2/q$$

If 'v' is the number of bits, the relationship between and 'q' are given by,

$$\langle q = 2^v \rangle$$

$$\left(S/N\right) = \frac{12P}{\left(\frac{2x_{max}}{q}\right)^2}$$

$$\left(S/N\right) = \frac{12Pq^2}{4x^2_{max}}$$

$$\left(S/N\right)_{normalized} = \frac{\overset{3}{\cancel{12}}Pq^2}{\cancel{4}_1}$$

For Normalized signal $x_{max} = 1$

$$\left(S/N\right) = 3Pq^2$$

$$\left(S/N\right) = 3P(2^v)^2 = 3P.2^{2v}$$

Now, to express (S/N) in decibels the above expression can be written as

$$\left(S/N\right)_{dB} = 10\log_{10}\left(S/N\right)_{dB}$$ where $P \leq 1$

$$= 10\log\left(3 \times 2^{2v}\right)$$

$$\left(S/N\right)_{dB} = 10\log 3 + 10(2v\log 2)$$

$$\left\langle \left(S/N\right)_{dB} = (4.78 + 6v)\ dB \right\rangle$$

Note :

- signalling rate / bit rate / bit per sample in PCM is $\langle r = vf_s \rangle$
- Transmission bandwidth of PCM is $(BW)_T \geq vW$
- average power of Quantization Noise $= \delta^2/12$
- Maximum signal to Noise ratio in PCM is

$$\left(S/N\right) = \frac{P}{\delta^2/12}$$

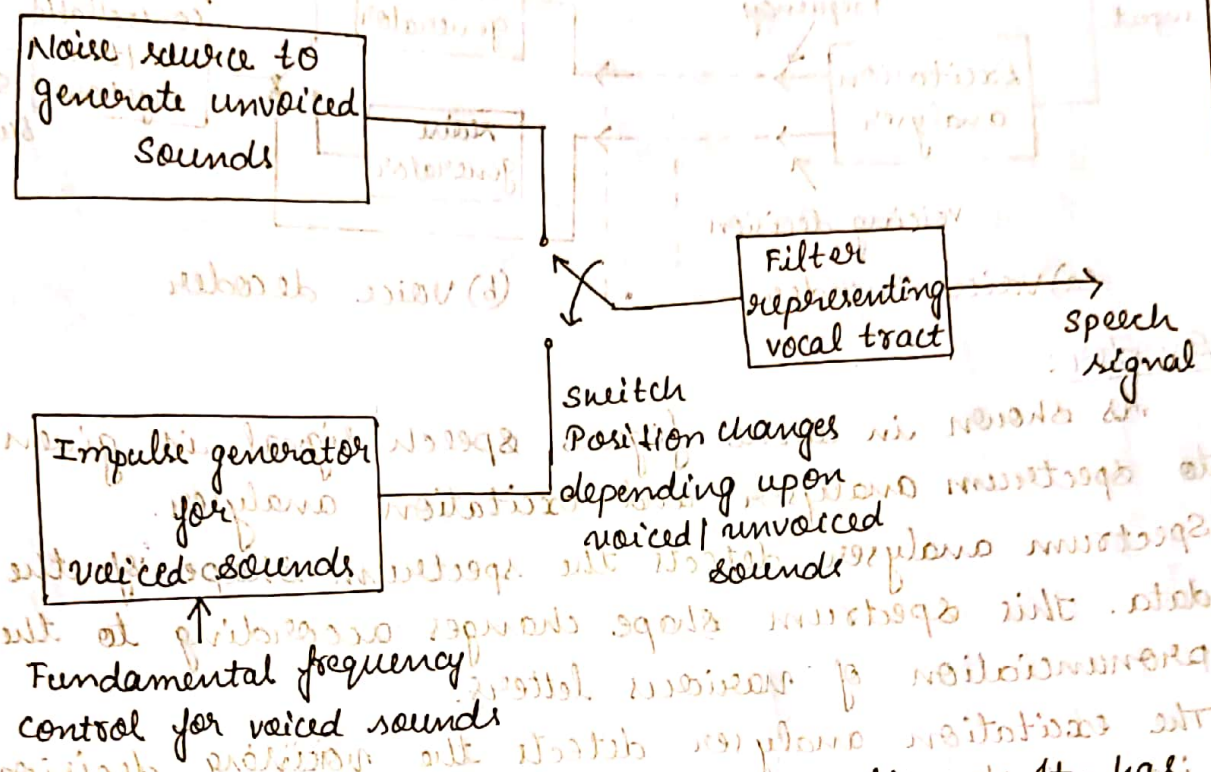
For $\left(S/N\right)_{normalized} = 3P.2^{2v}$

$$\left(S/N\right)_{dB} = (4.78 + 6v)\ dB$$

Application of vocoders:

The speech or voice coders are used for digital coding of speech. It is specially designed for coding speech signals. They operate at very low bit rate in the range of 1.2 to 2.4 kbps

Voice model:



Voice model is as shown in above figure. It has two frequency sources.

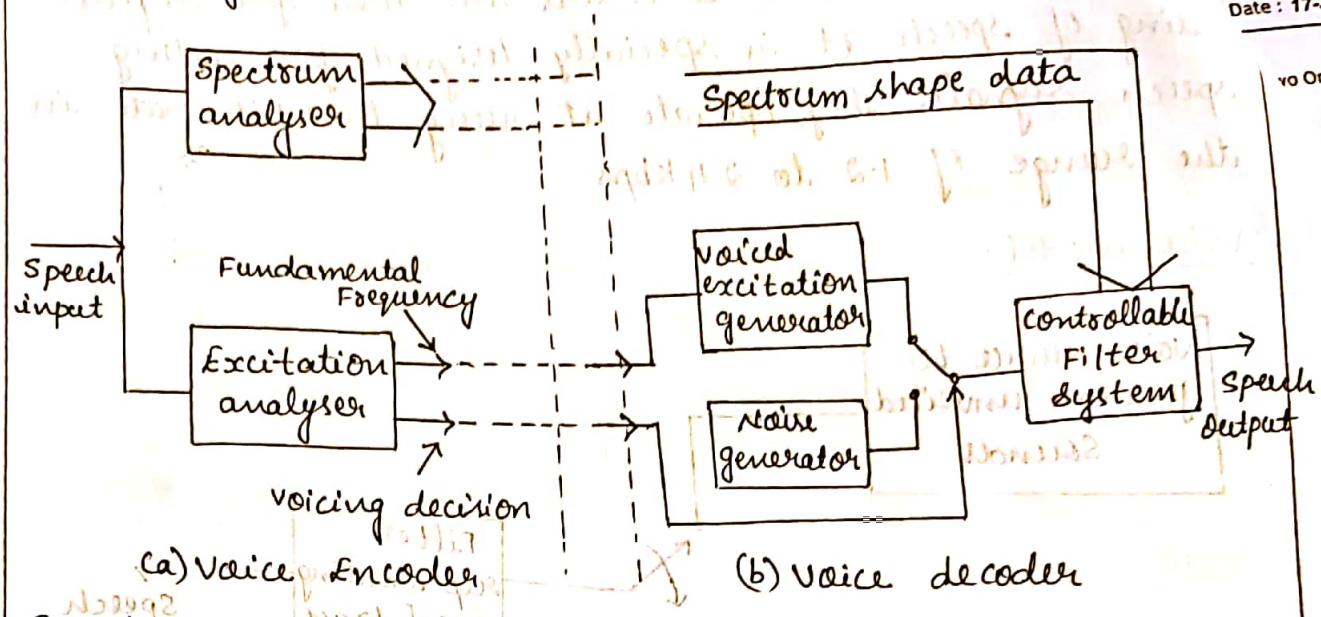
1) Frequency source used to generate unvoiced sounds when the speaker pronounces letter such as 's' & 'f'. A noise source is used to generate such unvoiced sound since their frequency spectrum is wide.

2) Voiced sound are simulated by impulse generator. Frequency is varied depending upon the pitch of the sound.

These voiced/unvoiced signals are then passed through a filter (vocal tract). In vocal tract, filtering is done with the help of tounge, lips, teeth etc.

Thus by using voice model, speech is generated.

Block diagram of vocoder:



Encoder:

As shown in above figure, speech signal is given to spectrum analyser and excitation analyser.

- Spectrum analyser detects the spectrum shape of the data. This spectrum shape changes according to the pronunciation of various letters.
- The excitation analyser detects the voicing decision i.e. whether speech is voiced/unvoiced. It also detects the fundamental frequency of voiced signals
- Thus the encoder converts speech signal in terms of fundamental frequency, voicing decision and spectrum shape.

Decoder:

The decoder is used to generate speech from the data given by encoder.

- Decoder contains noise generator and voiced excitat generator

frequency of voiced excitation generator is controlled depending upon fundamental frequency information from encoder.

- The voicing decision detects whether the sound is voiced / unvoiced.
- Controllable filter system, filters the signal generated from voiced excitation generator / noise generator.
- Frequency response of this filter is variable, it is changed according to spectrum shape data from the encoder. The output of the filter is the speech signal.